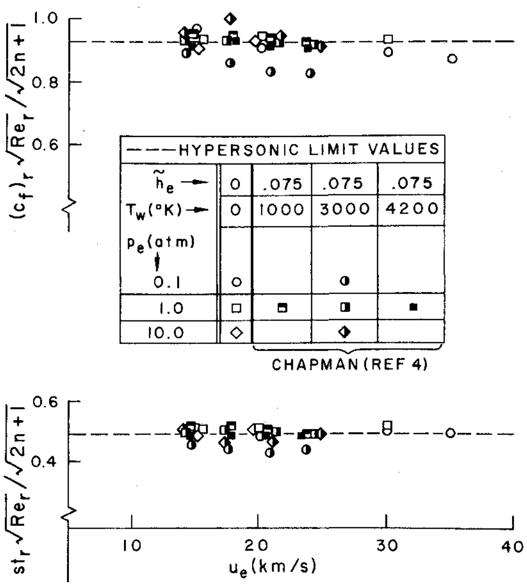

Fig. 2 Comparison of approximate and exact solutions for the friction and heat transfer parameters.

In the hypersonic limit, $\tilde{h}_w \rightarrow 0$ for a finite wall temperature and $\tilde{h}_e \rightarrow 0$ for a flat plate.†

Equations (7–10) have been solved for $\tilde{h}_e = \tilde{h}_w$ using approximations to the gas properties of the form

$$\tilde{\mu} = \tilde{h}^{\alpha}, \quad \tilde{\rho} = \tilde{h}^{-\beta}; \quad Pr_{eq} = \text{const} \tag{11}$$

For air at reentry flight conditions, values of $\alpha = 0.4, \beta = 0.8$ and $Pr_{eq} = 0.75$ were chosen using the results of Viegas and Howe.[2] The solutions were obtained by a finite-difference method as part of an investigation of radiating boundary layers,[3] and the results are shown in Fig. 1b. While the $(c_f)_e$ and st_e parameters continue to decrease monotonically with M_e, the $(c_f)_r$ and st_r parameters approach asymptotic limits with an error of less than 4% for $M_e > 10$. Using the parameter definitions, Eq. (11), and the hypersonic approximation $\tilde{h}_e = [(\gamma_e - 1)M_e^2/2]^{-1}$ gives the relationship

$$\frac{(c_f)_e Re_e^{1/2}}{(c_f)_r Re_r^{1/2}} = \frac{st_e Re_e^{1/2}}{st_r Re_r^{1/2}} = \left(\frac{\gamma_e - 1}{2}\right)^{-(\beta-\alpha)/2} M_e^{-(\beta-\alpha)} \tag{12}$$

Substitution in Eq. (12) of the values of α and β used previously, $\gamma_e = 1.4$, and the asymptotic values of $(c_f)_r Re_r^{1/2}$ and $st_r Re_r^{1/2}$ gives

$$\begin{aligned} (cf)_e Re_e^{1/2}/(2n+1)^{1/2} &= 1.28\, M_e^{-0.4} \\ st_e Re_e^{1/2}/(2n+1)^{1/2} &= 0.69\, M_e^{-0.4} \end{aligned} \tag{13}$$

as convenient, approximate formulas.‡

A comparison of these results with boundary layer solutions using the exact gas properties of Viegas and Howe[2] obtained by Dennar[3] and by Chapman[4] are shown in Fig. 2 where Chapman's results for the $(c_f)_e$ and st_e parameters have been converted to $(c_f)_r$ and st_r parameters. The agreement is good except for Chapman's results at $p_e = 0.1$ atm. When a check was performed on Chapman's calculations for this condition, the results for $(c_f)_r$ and st_r were within 6% and 2% respectively of the hypersonic limit values. The discrepancy between the two calculations is attributed to the different methods used to integrate the boundary-layer equations.

† When the hypersonic approximations $h_\infty = 0$ and $u_e = u_\infty \cos\theta$ are used, $\tilde{h}_e = \tan^2\theta$ where θ is the wedge or cone semi-vertex angle.

‡ Although the form of Eq. (13) is the same as that given by the "reference temperature" method, the numerical values are different.

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Determination of the Stiffness of an Elastic Bar

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Introduction

In Ref. 1 a method for nondestructive determination of the buckling load was proposed. The spring constants, which represent the actual boundary conditions, were obtained by measurements of the end deflections and rotations of the bar caused by a lateral load. The boundary conditions so obtained were inserted in the stability equations in order to predict buckling.

A different approach to the nondestructive testing method for the determination of the buckling load is presented in Refs. 2–5. The method is based on the observation that buckling loads of columns are raised if end restraint stiffnesses are increased while the flexibility of the member is decreased. The efforts are directed toward the establishment of empirical correspondence rules relating critical loads. As stated there the results obtained in this way are encouraging but much additional work remains to be done.

Although the method proposed in Refs. 2–5 is still impractical it poses a very attractive feature. A knowledge of

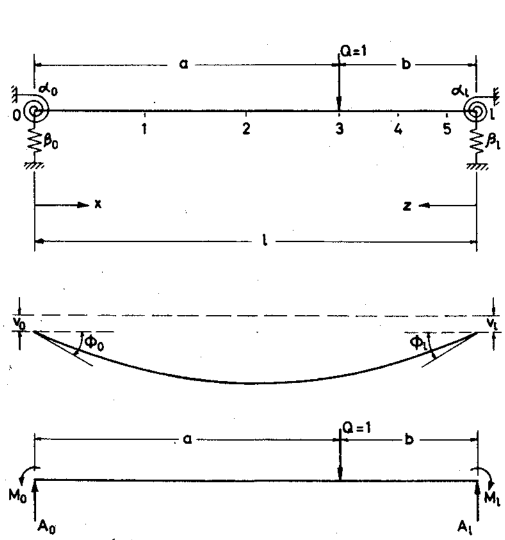

Fig. 1 Spring constrained beam loaded by a lateral force $Q = 1$.

Received March 5, 1971. This research was supported by Gerard Swope Foundation.

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the bending stiffness EI of the column is not required, whereas for the determination of the boundary conditions by the method proposed in Ref. 1, EI must be known a priori. Nevertheless by some modification of the method of Ref. 1 the bendingstiffness EI can be evaluated from some specific measurements.

In Ref. 1 the measurements of the deflections and rotations of the supports are needed, a requirement which is not always convenient. By a second modification of the method[6] it is found that the measurements may be performed at other, more convenient, points. Finally a third modification which combines the first two, is proposed.

Basic Equations

For convenience the equations of the elastic curve of a bar loaded by a lateral load $Q = 1$ are recapitulated briefly from Ref. 1, (See Fig. 1). By changing the necessary indices one obtains, for $0 \leq x \leq a$

$$EI(dv/dx) = M_0x - A_0(x^2/2) + EI\phi_0$$
$$EIv = M_0(x^2/2) - A_0(x^3/6) + EI\phi_0x + EIv_0 \tag{1}$$

for $0 \leq z \leq b$

$$EI(dv/dz) = M_lz - A_l(z^2/2) + EI\phi_l$$
$$EIv = M_l(z^2/2) - A_l(z^3/6) + EI\phi_lz + EIv_l \tag{2}$$

Equilibrium conditions

$$A_0 + A_l = Q, \quad M_0 - A_0a - M_l + A_lb = 0 \tag{3}$$

Compatibility conditions

$$(dv/dx)_{x=a} = -(dv/dz)_{z=b}, \quad (v)_{x=a} = (v)_{z=b} \tag{4}$$

Determination of the Boundary Conditions

A. Measured values ϕ_0, v_0, v_3, ϕ_l, v_l: Bending stiffness E, unknown.

By addition of one more measured value than in Ref. 1, say the deflection of point 3, one can evaluate not only the reaction forces and moments but also the bending stiffness of the bar. From the basic equations one obtains:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ a & -b & -1 & 1 & 0 \\ a^2/2 & b^2/2 & -a & -b & -(\phi_0 + \phi_l) \\ a^3/6 & -b^3/6 & -a^2/2 & b^2/2 & [v_l - v_0 + (\phi_lb - \phi_0a)] \\ a^3/6 & 0 & -a^2/2 & 0 & [v_3 - (\phi_0a + v_0)] \end{bmatrix} \begin{Bmatrix} A_0 \\ A_l \\ M_0 \\ M_l \\ EI \end{Bmatrix} = \begin{Bmatrix} Q = 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{5}$$

B. Measured values: v_1, v_2, v_4, v_5: Bending stiffness: EI, known[6]

Sometimes it is more convenient to perform measurements at points which do not belong to the supports of the beam and then, the end deflections and rotations ϕ_0, v_0, ϕ_l, v_l will be unknown. However, by measuring the deflections at points 1, 2, 4 and 5 using the basic equations one obtains

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ a & -b & -1 & 1 & 0 & 0 & 0 & 0 \\ a^2/2 & b^2/2 & -a & -b & -1 & -1 & 0 & 0 \\ a^3/6 & -b^3/6 & -a^2/2 & b^2/2 & -a & b & -1 & 1 \\ -x_1^3/6 & 0 & x_1^2/2 & 0 & x_1 & 0 & 1 & 0 \\ -x_2^3/6 & 0 & x_2^2/2 & 0 & x_2 & 0 & 1 & 0 \\ 0 & -z_4^3/6 & 0 & z_4^2/2 & 0 & z_4 & 0 & 1 \\ 0 & -z_5^3/6 & 0 & z_5^2/2 & 0 & z_5 & 0 & 1 \end{bmatrix} \begin{Bmatrix} A_0 \\ A_l \\ M_0 \\ M_l \\ EI\phi_0 \\ EI\phi_l \\ EIv_0 \\ EIv_l \end{Bmatrix} = \begin{Bmatrix} Q = 1 \\ 0 \\ 0 \\ 0 \\ EIv_1 \\ EIv_2 \\ EIv_4 \\ EIv_5 \end{Bmatrix} \tag{6}$$

C. Measured values v_1, $dv_2/dx = \phi_2$, v_3, v_4, v_5: Bending stiffness EI, unknown

From the examples given previously it is clear that when the bending stiffness EI is unknown it is necessary to have five independent measured values whereas when EI is known there are needed only four. The measured values may be deflections or rotations. If deflections and rotations are used they may be taken at the same point. The point at which the measurements are performed need not necessarily be arranged as shown in Fig. 1 but then the relevant basic equation must be used. For example if the measured values are the deflection of point 1, the rotation of point 2 and the deflections of points 3–5, the basic equations yield

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a & -b & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ a^2/2 & b^2/2 & -a & -b & -1 & -1 & 0 & 0 & 0 \\ a_3/6 & -b^3/6 & -a^2/2 & b^2/2 & -a & b & -1 & 1 & 0 \\ x_1^3/6 & 0 & -x_1^2/2 & 0 & -x_1 & 0 & -1 & 0 & v_1 \\ x_2^2/2 & 0 & -x_2 & 0 & -1 & 0 & 0 & 0 & \phi_2 \\ a^3/6 & 0 & -a^2/2 & 0 & -a & 0 & -1 & 0 & v_3 \\ 0 & z_4^3/6 & 0 & -z_4^2/2 & 0 & -z_4 & 0 & -1 & v_4 \\ 0 & z_5^3/6 & 0 & -z_5^2/2 & 0 & -z_5 & 0 & -1 & v_5 \end{bmatrix} \begin{Bmatrix} A_0 \\ A_l \\ M_0 \\ M_l \\ EI\phi_0 \\ EI\phi_l \\ EIv_0 \\ EIv_l \\ EI \end{Bmatrix} = \begin{Bmatrix} Q = 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{7}$$

Again, as stated in Ref. 1, better average evaluations of the measured deflections and rotations will be obtained by changing the value of Q and dividing the measured values by it and better statistical values for the spring constants will be obtained by changing the position of Q.

The spring constants must be evaluated by the relations

$$\alpha_0 = M_0/\phi_0, \quad \beta_0 = A_0/v_0, \quad \alpha_l = M_l/\phi_l, \quad \beta_l = A_l/v_l \tag{8}$$

The actual boundary conditions of Eq. (8) obtained are inserted in the stability equations and the buckling load is evaluated.[1] It must be noted that this buckling load is the ideal elastic buckling load, in which no eccentricity and no possibility of plastic behaviour of the material are taken into account.

It seems that the proposed method will work also when the bending stiffness is not constant but slightly varying. In this case the bending stiffness obtained will be some average generalized bending stiffness.

Conclusion

A method for the determination of the boundary conditions of an elastic bar was described. The measured values may be performed at points which are not necessarily the supports of the bar. The bending stiffness of the bar is not necessarily known.

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Absolute Velocity Determination in a Hypersonic Low-Density Flow

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An accurate knowledge of the absolute velocity in a hypersonic low-density tunnel would provide a useful check on the usual methods of stagnation temperature determination above 1400°K, which is based on a comparison of the actual mass flux and heater power with room temperature values. Furthermore, using diatomic nitrogen as a test gas, a definite knowledge of the flow velocity gives some information about the extent of vibrational relaxation. Initial attempts to measure the flow velocity utilize mass flux probes, but the necessity of applying pitot pressure corrections introduces uncertainties.[1]

The described technique for velocity determination is based on time-of-flight measurements of nitrogen ions produced by a short pulsed high energy electron beam.

The following investigations have been carried out in the hypersonic low-density wind tunnel of the AVA Göttingen (West Germany). The stagnation temperature was varied between 600 and 2250°K at a stagnation pressure of 50 atm absolute. The Mach number in the test region was about 20. At stagnation temperatures below 1200°K condensation of nitrogen during the expansion was expected without greatly affecting the velocity measurements.

The stagnation temperature was determined by comparison of the total mass flux of the test gas under experimental and room temperature conditions with the use of carefully measured calibration curves.

To monitor the temperature measurements, the heater power input was observed. A decrease in mass flux at constant power input indicated a change in nozzle geometry and thus rendered the starting conditions of a run invalid.

Received February 22, 1971; revision received April 5, 1971.

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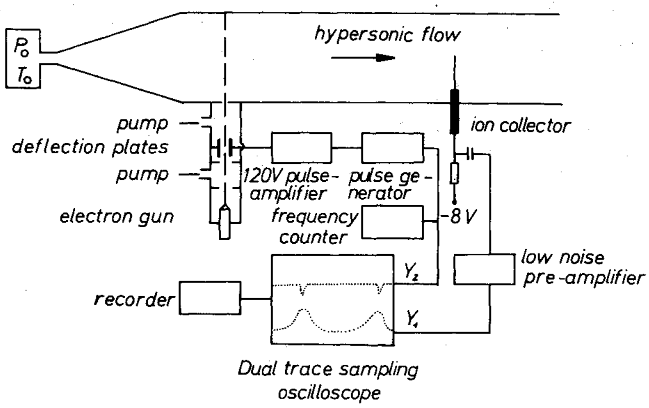

Fig. 1 Schematic drawing of the experimental setup.

The experimental setup is shown in Fig. 1. A 2-μsec pulse of the electron beam (20 KeV, 200 μA) produced ions within a small volume defined by the beam diameter and the beam length across the test region. With the basic assumption of negligible transfer of momentum from the fast electrons to the relative slow molecules, changes of the flow bulk velocity because of the electron beam interaction do not occur. At a distance of 375-mm downstream of the electron beam, the ions were gathered by the ion collector, which consists of a simple isolated tungsten wire of 0.1-mm diameter and 30-mm length suspended parallel to the electron beam. The starting pulse of the electron beam and the amplified ion signal were recorded by a dual trace sampling oscilloscope. The time-of-flight was determinated by varying the repetition frequency of the starting pulse in such a manner, that the $(n + 1)$th starting pulse coincided with the peak value of the nth ion signal. The maximum of the ion signal was used as the mean time of arrival of the ions. The repetition frequency was measured by a quartz stabilized frequency meter. The result could be reproduced within an error of about 0.2%. Using the scanning outputs of the oscilloscope the starting pulse and the ion signal could be recorded.

Since the flight path is known (within an error of ±0.3%) the flow velocity was found as the product of measured fre

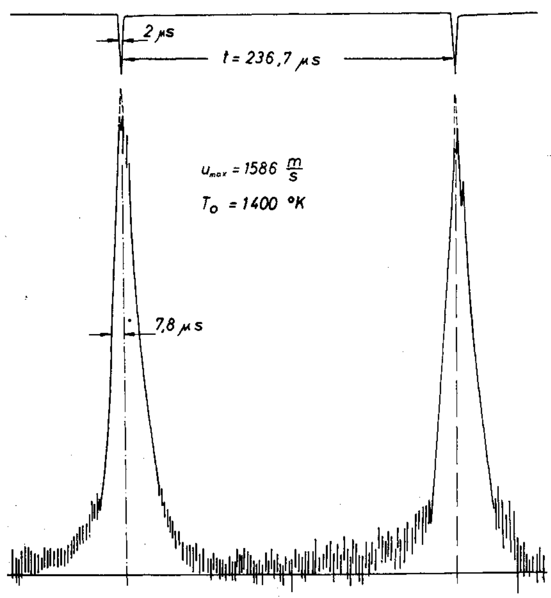

Fig. 2 Starting pulse (upper trace) and ion signal (lower trace).